On the visualization of curvature

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Abstract
The first step towards creating high-quality models is to discover if any unevenness is present on the surfaces. Curvature maps have long been used for this purpose, but their conventional heat map visualization does not do justice to the sensitivity this tool can provide. Here we propose a simple alternative that helps highlighting problems with G^3-continuity.

1. Introduction

Generating aesthetically pleasing models is an important topic of computer-aided design, especially in the automotive industry. 'Class-A' surfaces are notoriously hard to create. Traditionally, car bodies are first built from clay and manually modified by designers, then a 3D scan is taken that serves later as a basis for surface creation.[3] Production lines usually have a section illuminated with parallel neon lights to check for jumps and wiggles in the reflections (see Fig. 1).

Modeling systems offer several *interrogation tools* for checking surface quality. The simplest method is *contouring*, where the model is sliced by parallel planes. The smoothness of the intersection curves is a good indicator of the quality of the whole model. Often the curvature comb of these curves is displayed, as well. Contouring is very robust, as it depends only on positional data, but contour lines can only show errors in G^1 continuity (where the lines have a cusp).

Figure 1: Checking the reflection lines ('cubing'). Image from Sußner et al.[5]

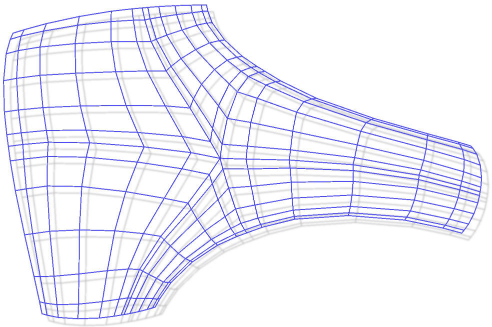
Reflection lines are expensive, but the easily computable *isophotes* or *zebra maps*, or the *highlight band*[2] have very similar properties. Since these depend on the normal vectors, they are able to show errors in G^2 continuity (where the lines have a cusp), as well as G^1 artefacts (where the lines do not meet). Moving the reference point and looking at the flow of isophote lines can also give an indication of higher-order problems.

Cubic B-spline curves and surfaces, which have only C^2 continuity at the knot positions, are the *de facto* standard in the world of CAD, but the importance of the *variation* of curvature (G^3) was recognized early on.[4] Modern guides even recommend modeling with quintic curves and surfaces, and discourage the use of interior knots.[1]

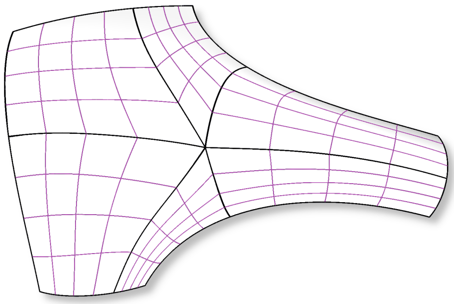
Mean and Gaussian curvatures, as second-order quantities, should be able to point out errors in G^3. However, they are conventionally visualized as smoothly colored heat maps, which makes it hard to discern problems in surface quality. In the rest of the paper, we will look at an alternative visualization of curvature that arguably fares better in this respect.

2. Quantized and striped curvature

Our running example will be a collection of cubic B-spline patches with 3 internal knots in both parametric directions, see Figure 2. The individual surfaces are connected with exact C^0, but only approximative G^1/G^2 continuity. Normals

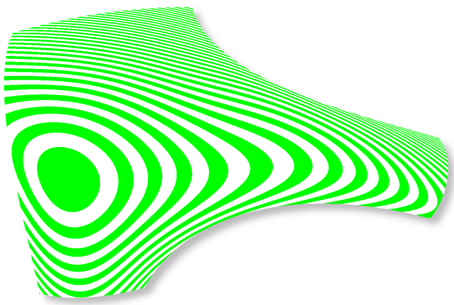


(a) Control nets

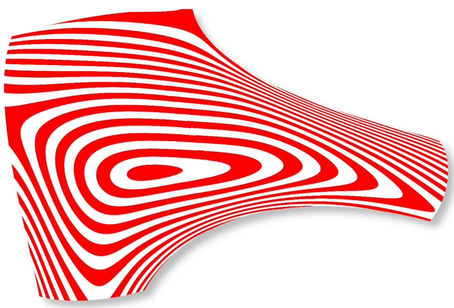


(b) Patch boundaries (black) and knot lines (purple)

Figure 2: A collection of cubic B-spline patches.

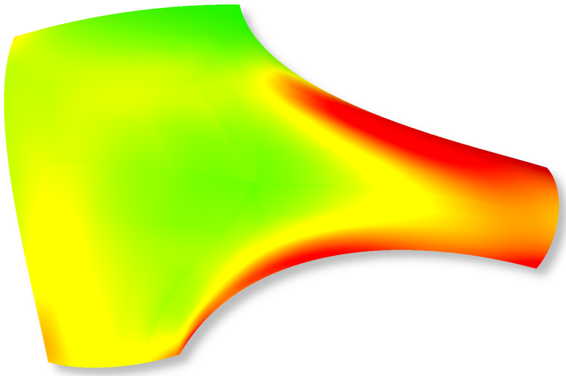


(a) Contouring (‘slicing’)

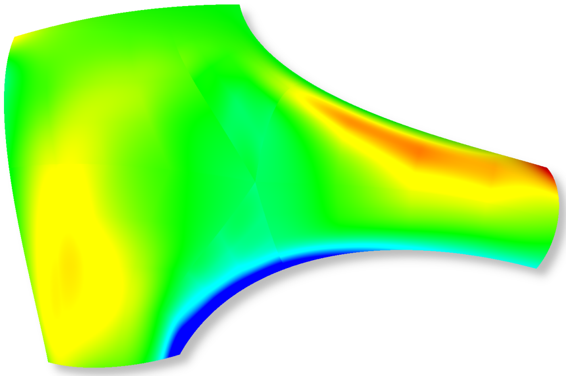


(b) Isophote lines

Figure 3: Stripe-based interrogation methods.



(a) Mean curvature map



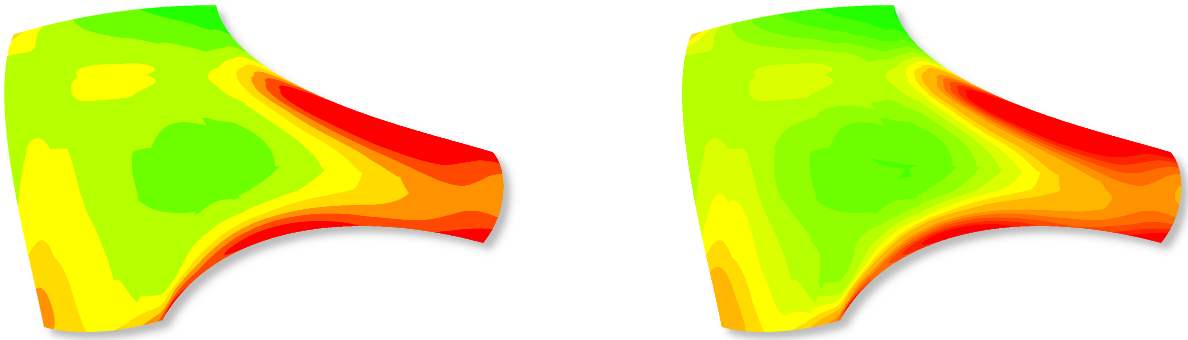
(b) Gaussian curvature map

Figure 4: Curvature maps.

and curvatures are evaluated *exactly* on a dense mesh. Both the contours (Fig. 3a) and the isophote lines (Fig. 3b) look very smooth, and the curvature maps (Fig. 4) are quite nice, as well. Only the Gaussian map shows some unevenness, but even those are hardly detectable. The color scale is set based on a user-specified interval $[\min, \max]$ (default is the range between the 5^{th} and 95^{th} percentiles): values below min are blue, values above max are red, and zero curvature is green. The hue of values in-between is linearly interpolated.

The problem is that small changes in the colors are hard to percieve with the human eye. We can enhance the image by using *fewer* colors – a quantized heat map, as shown in Figures 5–6. The problems are much more easily distinguishable now.

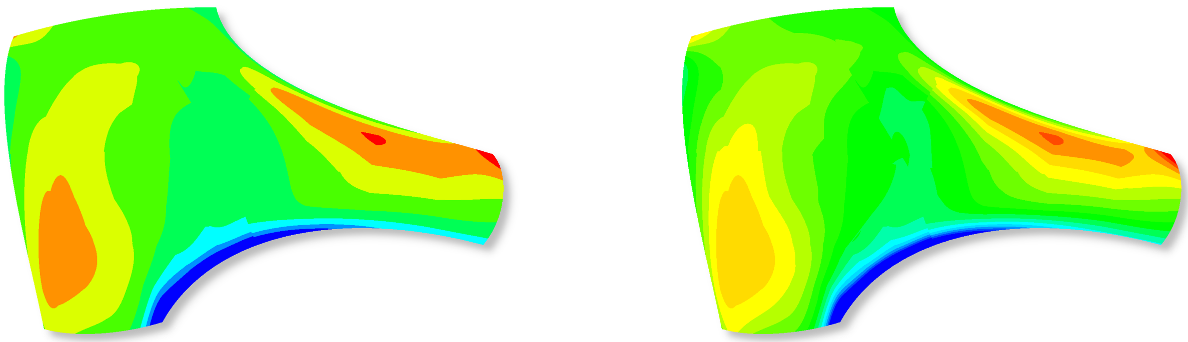
Still, we can do better. Using a bicolor map, like in the case of contouring and isophote lines, gives the most pronounced effect, see Figure 8. The right-hand side images also show the boundaries and knot lines, which coincide with the artefacts (non-matching lines and cusps for G^2 and G^3 errors, respectively) visible on the left-hand side images. Note that in this case we can use a repeating texture, so there is



(a) 16-color scale

(b) 32-color scale

Figure 5: Quantized mean curvature.



(a) 8-color scale

(b) 16-color scale

Figure 6: Quantized Gaussian curvature.

a color change corresponding to a change of $(\max - \min)/n$ in curvature, even outside the specified range.

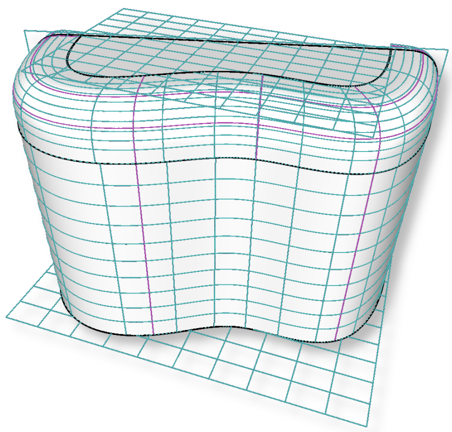


Figure 7: Edge blend model with boundaries (black), isolines (turquoise) and knot lines (purple).

The results look so much worse that it is natural to ask: can these striped curvatures be nice at all? To show that this can indeed be done, we will use another example—an edge blend between a quartic-by-linear spline and a (trimmed) plane, shown in Figure 7. The blend itself is quintic in the transversal direction. Both the mean curvature (Fig. 9) and the Gaussian curvature (Fig. 10) look very smooth.

Conclusion

In conclusion, we have seen that just by quantizing the curvature and using a bicolor map, G^3 problems become easily distinguishable. We recommend using this technique when creating aesthetic surfaces, possibly in combination with traditional heat maps, which can also show information on the curvature values.

Acknowledgements

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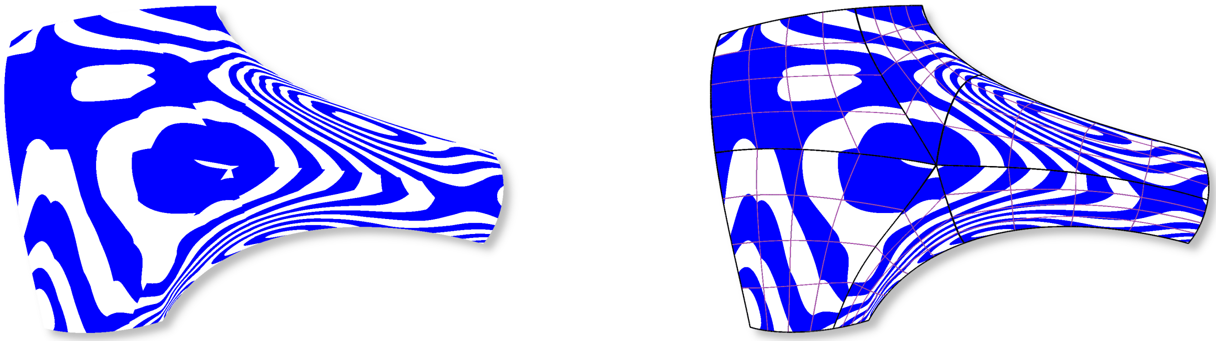
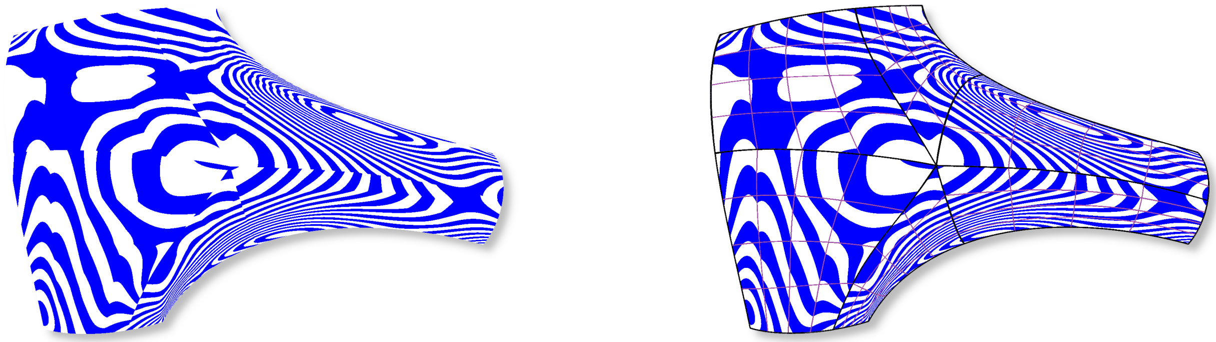
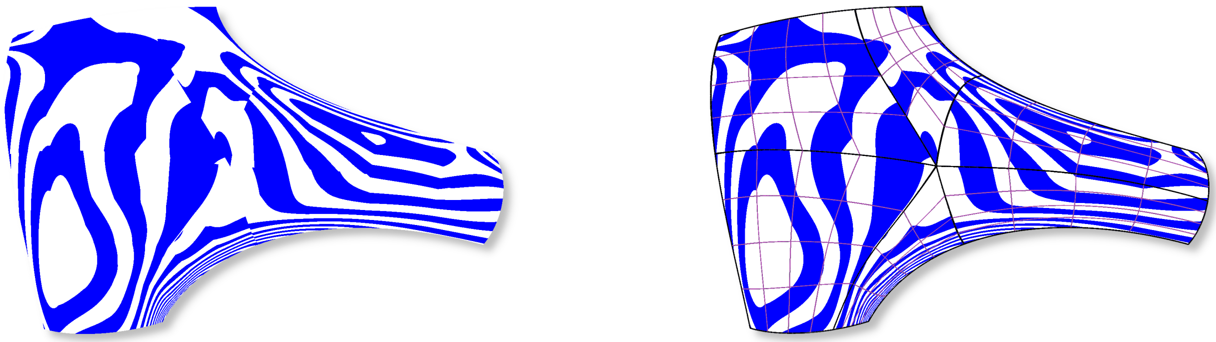


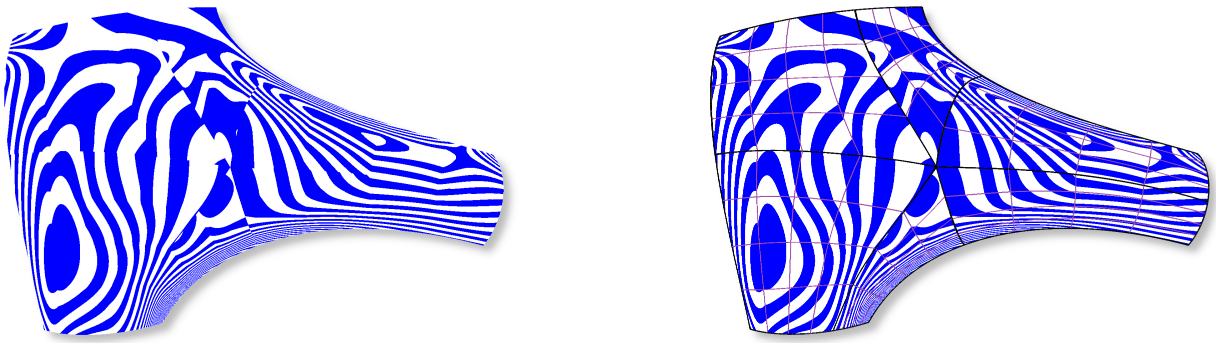
(a) Mean curvature, $n = 16$



(b) Mean curvature, $n = 32$



(c) Gaussian curvature, $n = 8$



(d) Gaussian curvature, $n = 16$

Figure 8: Striped curvatures (without and with boundaries and knot lines).

Figure 9: Striped mean curvature of the edge blend model.

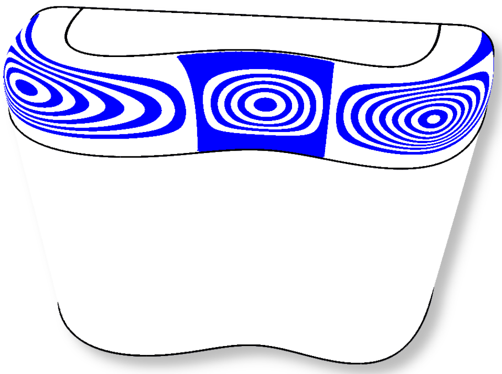


Figure 10: Striped Gaussian curvature of the edge blend model.

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